The Dynamics Of Convergence In The Context Of A Balanced Growth Path

By

Azmi Shahrin Bin Abdul Rahim

Nobel Laureate Robert M. Solow expounded his model in the article "A Contribution To The Theory of Economic Growth" which was published in the February 1956 issue of the Quarterly Journal Of Economics. In this article, he had elegantly demonstrated the dynamics of economic growth which tended to converge to a stable balanced path which was in contrast with the earlier Harrod-Domar model which in Solow's own words was "at best balanced on a knife-edge of equilibrium growth" and hence unstable.

Solow's explication of the Theory of Economic Growth assumed a closed economy without a Government sector, which produced only one commodity under constant returns to scale which was represented by the technical possibilities frontier Y= F(K,L), where K and L stood for Capital stock and Labour force, respectively[1]. Labour was fully employed at the frontier. Output, which was not consumed, was saved and invested. Hence, total savings was sY where s was the constant savings rate. The rate of change in Capital stock dK/dt was simply sY in which depreciation was implicit. Therefore,

Equation (1) $$\frac{dK}{dt} = \dot{K} = sY = sF(K, L)$$

Further, it was assumed that Labour force grows exogenously at the constant rate n, so that Labour force at any instant was

Equation (2) $$L(t) = L_0 e^{nt}$$

Equation (1) was made more complete by using Equation (2).

Equation (3) $$\dot{K} = sF(K, L_0 e^{nt})$$

Now, the following Capital Labour ratio was introduced

[1] David Romer in his popular graduate school text Advanced Macroeconomics (1996), expanded Solow's original equation by augmenting labour with knowledge and making depreciation explicit in the model.

Equation (4) $r = \frac{K}{L}$

So that

Equation (5) $K = rL = rL_0e^{nt}$

K was differentiated with respect of time, which yielded the following equation

Equation (6) $\frac{dK}{dt} = \dot{K} = L_0e^{nt}\dot{r} + nrL_0e^{nt} = L_0e^{nt}(\dot{r} + nr)$

Equating the 2 identities for dK/dt in Equations (1) and (6) gave the following

Equation (7) $sF(K, L_0e^{nt}) = L_0e^{nt}(\dot{r} + nr)$

L_0e^{nt} was divided on both sides of Equation (7) and then rearranged to this defining equation

Equation (8) $\dot{r} = sF(r,1) - nr$

This implied that the rate of change in the capital labour ratio with respect to time was dependant upon total savings and the labour force growth rate, which was the first, and the second term respectively on the right hand side of Equation (8). Romer described nr as the breakeven investment level required to keep up with labour growth. The following equations described this relationship.

Equation (9) $\dot{r} > 0$ when $sF(r,1) > nr$, $\dot{r} < 0$ when $sF(r,1) < nr$, $\dot{r} = 0$ when $sF(r,1) = nr$

Equation (8) could also be restated so that

Equation (10) $$\dot{r} = sF(r,1) - nr = r\frac{sF(K,L)}{K} - nr = \frac{\dot{K}}{K}r - \frac{\dot{L}}{L}r = \left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right)r$$

This meant that the difference between the growth rates of Capital and Labour determined $\dot{r}$ which could be described as follows:

Equation (11) $$\dot{r} > 0 \text{ when } \frac{\dot{K}}{K} > \frac{\dot{L}}{L},\ \dot{r} < 0 \text{ when } \frac{\dot{K}}{K} < \frac{\dot{L}}{L},\ \dot{r} = 0 \text{ when } \frac{\dot{K}}{K} = \frac{\dot{L}}{L}$$

Solow in his article gave 2 examples which illustrated the dynamics of convergence in a balanced growth path. The first of these would be Figure 1.

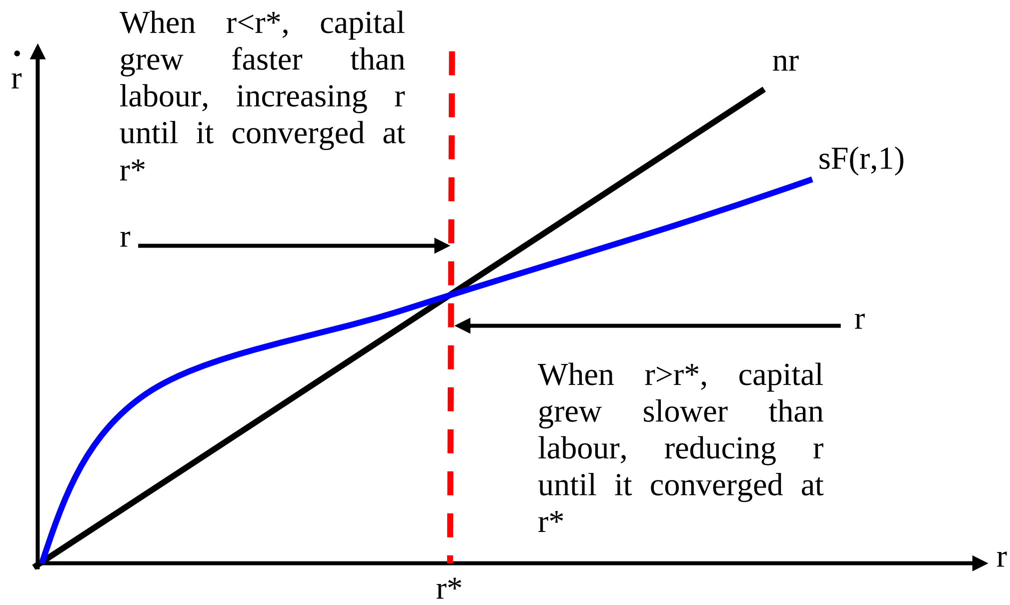

Figure 1: The Dynamics of Convergence

The balanced growth path was the unique point r* where sF(r,1)=nr. At this point, $\dot{r} = 0$ the economy was at equilibrium because the growth rate of capital was on par with the growth rate of labour. This was a stable point and would be maintained because capital and labour would continue to grow in this same ratio. Output (the economy) would grow at the same rate as labour due to the assumption of constant returns to scale while productivity of labour remained constant.

To the left of r* (except the origin), $\dot{r} > 0$ because sF(r,1)>nr which meant that capital stock was growing faster than labour. This increased r until it converged to the equilibrium r*. To the right of r*, $\dot{r} < 0$ because sF(r,1)<nr which meant that labour was growing faster than capital stock. This reduced r until it converged to the equilibrium r*. In both occasions upon reaching equilibrium r*, this capital labour ratio would be maintained thereafter. The equilibrium was a stable state because r converged to r*, regardless of the initial capital labour ratio.

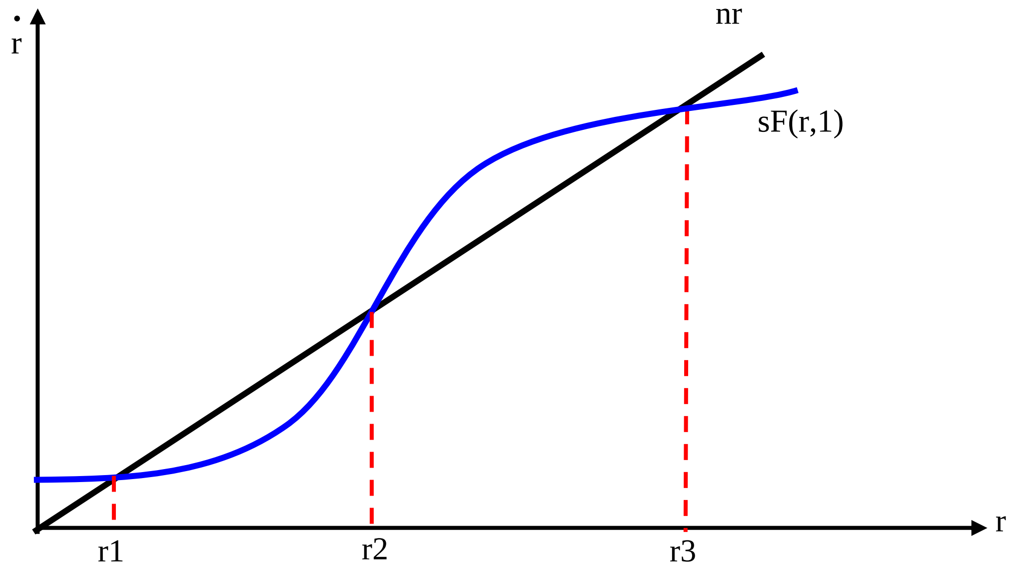

Figure 2: Another Model Of Convergence

Figure 2 followed the dynamics illustrated earlier where r converged to r1 if the initial capital labour ratio was between 0 and r2. It would however converge to r3 for any intial capital labour ratio exceeding r2. The capital labour ratio was stable at the r1 and r3 equilibriums. Here, the capital labour ratio capital stock and output would expand at the growth rate of labour, which was n. However, productivity of labour at r1 was less than r3 because r1 had less capital stock than r3. The economy might settle at r2, which was also equilibrium. However, r2 was not a stable equilibrium because it could be easily upset. Therefore, in this example the capital labour ratio also converged to the equilibrium of a balanced growth path.

However, Solow also demonstrated an example using Figure 3 where r would not converge to equilibrium.

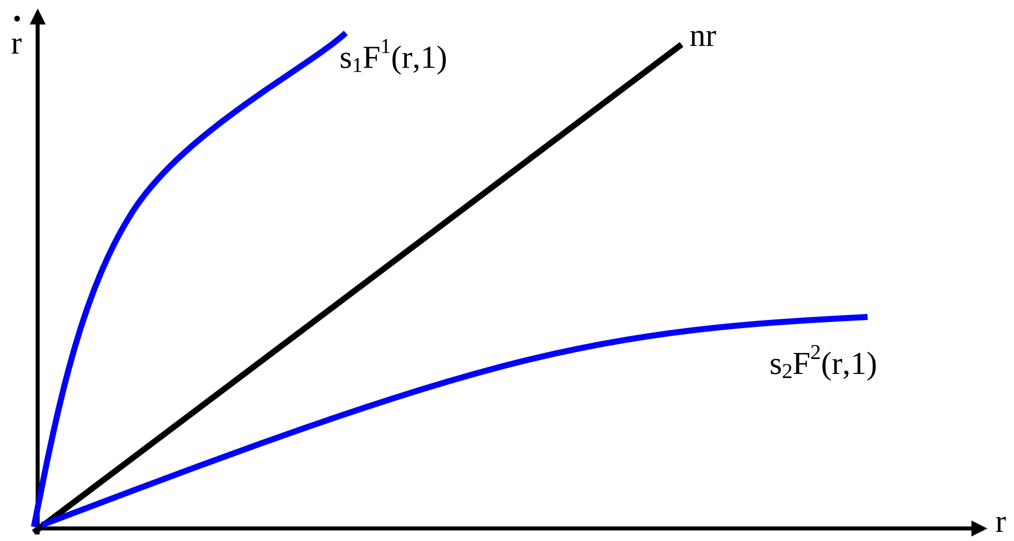

Figure 3: A Model Of Non Convergence

This example $s_1F^1(r,1)$ laid wholly above nr. It was so productive that ever-increasing output increased r ad infinitum because growth of capital stock exceeded growth of labour. Meanwhile, $s_2F^2(r,1)$ was always below nr because capital growth rate never exceeded labour growth rate. In the first instance r, capital and output increased perpetually without ever converging towards equilibrium. In the second instance r, capital and output decreasingly converge towards the origin. Consequently, the economy would never be on a balanced growth path in both instances.

In conclusion, whether the capital labour ratio converged to a stable equilibrium and a balanced growth path depended solely on the specific condition sF(r,1)=nr. The ratio would converge if and only if this condition exist. If this condition did not exist, then r could possibly increase ad infinitum without convergence